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Ph.D. DISSERTATION DEFENSE

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Date:	Apr 1st
Time/Location:	3:45pm at North building 316
Title:	One-variable equations over the lamplighter group
Chairperson:	Alexander Ushakov, Charles V. Schaefer, Jr. School of Engineering and Science / Department of Mathematical Sciences
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ABSTRACT

We study one-variable equations over the lamplighter group $\MZ_2 \wr \MZ$. While the decidability of arbitrary equations over L_2 remains open, we prove that the Diophantine problem for single equations in one variable is decidable. Our approach reduces the problem to a divisibility question for families of parametric Laurent polynomials over $\MZ_2$, whose coefficients depend linearly on an integer parameter. We develop an automaton-theoretic framework to analyze divisibility of such polynomials, exploiting eventual periodicity phenomena arising from polynomial division over finite fields. This yields an explicit decision procedure, which is super-exponential in the worst case. On the other hand, we show that for a generic class of equations, solvability can be decided in nearly quadratic time. These results establish a sharp contrast between worst-case and typical computational behavior and provide new tools for the study of equations over wreath products.