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Ph.D. DISSERTATION DEFENSE

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Date: Wednesday, April 23rd, 2025
Time/Location: 10:30 am, Howe Center 303
Title: Advances in Risk Measure Theory and Statistical Estimation for Composite Functionals in Stochastic Optimization

Chairperson: Darinka Dentcheva, Department of Mathematical Sciences, School of Engineering & Sciences

Committee Members: Brendan Englot, School of Engineering and Sciences
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ABSTRACT

This work focuses on the theory and statistical estimation of risk measures that may exhibit nonlinearity in probability. This topic has become increasingly central to recent advancements in stochastic optimization. A broad class of such risk models can be expressed as composite risk functionals, whose analysis is facilitated by embedding their structure into a functional space.

Our initial objective is to establish central limit theorems for composite risk functionals under mixed estimation schemes. We begin by analyzing vector-valued functionals and develop a framework for evaluating high-dimensional risks. This enables the comparison of multiple risk measures and supports the estimation and asymptotic analysis of systemic risk and its optimal value in decision-making. We then derive new central limit theorems for optimized composite functionals using mixed estimators, including empirical and smoothed variants. Verifiable conditions for these limit theorems are provided, and we demonstrate their applicability to a range of risk measures. Furthermore, we construct more accurate confidence intervals for these functionals, utilizing not only central limit approximations but also tools from large deviations theory and Rademacher complexity to derive sharper probabilistic bounds.

A separate line of research in this work introduces smoothed risk forms and associated smoothed mini-batch. We first show that coherent and law-invariant risk forms generate smoothed counterparts that preserve these desirable properties. We further derive dual and Kusuoka representations of the smoothed risk form. Our analysis then focuses on Gaussian-smoothed risk forms, particularly the bias and concentration inequalities between the Gaussian-smoothed risk form and its mini-batch counterpart. These forms demonstrate significantly faster convergence rates, which can be used to improve the accuracy of risk evaluation.