SAMPLE PROBLEMS FROM THE STEVENS MATH OLYMPIADS

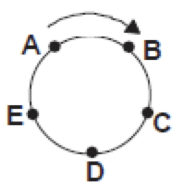
1. Sample problems

Below are four sample problems per division of the Stevens Math Olympiad, held annually at Stevens Institute of Technology in Hoboken, New Jersey. The sample problems are taken from the Olympiads held in 2016, 2017, 2018, and 2019. Answers are given on the last page.

Note that Olympiad problems are intended to be challenging! Challenging problems both contribute to the spirit of a mathematics competition and help us single out winners from a large pool of participants. Solving even a handful of problems at the Olympiad (each student will be offered 15 problems) is a result to be proud of.

Grades 3-4

(1) An ant travels around a circle in the direction shown. As it moves, it touches each of the labeled points in order. The first three points that the ant touches are A, B, and C, in that order. What is the 28th point that the ant touches?



(2) Camila is hanging upside-down from the monkey bars at the playground. If her left ear is facing east, then what direction is her nose facing?

(3) A rectangle has a perimeter of 2 meters and a length of 70 centimeters. Find the area of the rectangle in square centimeters.

(4) A worker must paint a fence made out of vertical wooden poles. Whenever two poles are such that 2 or 3 other poles are in between them, the two poles are required to be painted in different colors. What is the smallest number of colors the worker can use to paint the fence?

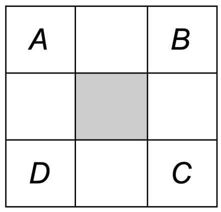
Grades 5-6

(1) Nicole has three times as many stickers as Sharon. Sharon has twice as many stickers as Ariel. If Ariel has fewer than 8 stickers, then what is the greatest number of stickers that Nicole can have?

(2) The denominators of two fractions are consecutive natural numbers. Both fractions are in lowest terms, and their sum is $\frac{51}{56}$. Find the greater of the two fractions.

(3) When a four-digit number is divided by 3 or by 7, the remainder is 1. A Stevens professor doubles this number, then writes the result on a whiteboard. What is the smallest possible value of the number written by the professor?

(4) Consider the 3×3 grid (with the middle square removed) shown below. The odd numbers from 1 to 15 are placed in the squares (one number per square) so that the top row adds up to 19, the bottom row adds up to 35, the left column adds up to 19, and the right column adds up to 27. What is the value of $A + B + C + D$?



Grades 7-8

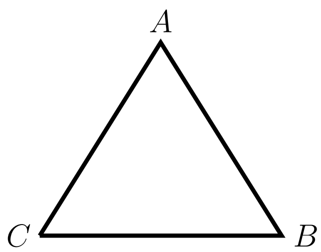
(1) Each of 25 cards is labeled either with the number 3 or 4. The sum of all of the numbers on the cards is 88. How many cards are labeled with the number 3?

(2) In the addition problem

$$\text{STEVENS} + \text{HOBOKEN} = 9495753,$$

each letter stands for one of the digits $0, 1, \ldots, 9$. The same letter always stands for the same digit, and different letters stand for different digits. Assuming the addition is correct, what four-digit number does the word HENS represent?

(3) A particle is located at vertex A of the triangle ABC shown below. Suppose that the particle randomly takes a step along one of the edges of the triangle, with equal probability assigned to each adjacent vertex. What is the probability that the particle will first return to vertex A after 5 or more steps?



(4) Assuming the equation $ax^3 + bx^2 + cx + d = 0$ has no real roots and $b + d > c$, find the sign of b and the sign of d.

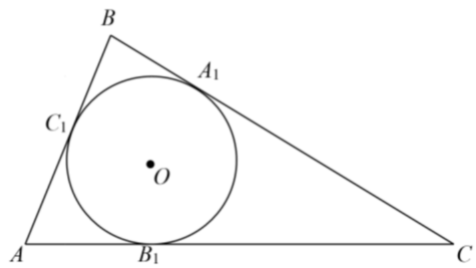
Grades 9-10

(1) Find the lowest multiple of 7 such that dividing it by any of the numbers $2, 3, 4, 5,$ and 6 yields a remainder of 1.

(2) Two different numbers x and y, not necessarily integers, satisfy the equation $x^3 - 1870x = y^3 - 1870y$. Find the value of $x^2 + xy + y^2$.

(3) Two riders on horseback simultaneously leave Village A. They proceed with different but constant speeds to Village B and then return without stopping. On her way back to Village A, one of the riders overtakes the other, meeting her at a point located m miles from Village B. Upon returning to Village A, she leaves for Village B again and again meets the other rider after covering one third of the distance between Village A and Village B. Find the distance between the two villages.

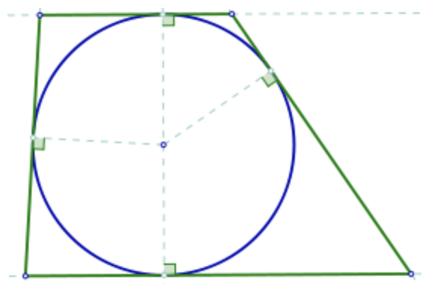
(4) Let O be the center of the circle inscribed in the triangle ABC (see the figure below). Let A_1, B_1, and C_1 be the points of tangency with the sides BC, AC, and AB, respectively. Let O_1, O_2, and O_3 (not shown) be the centers of the circles circumscribed around the triangles OB_1C_1, OA_1C_1, and OB_1A_1, respectively. Find the area of the triangle $O_1O_2O_3$ if the area of the triangle ABC is 24.



Grades 11-12

(1) When divided by 3, 6, and 12, a number has remainders R_1, R_2, and R_3, respectively. If the sum of the three remainders is equal to 15, what is R_1?

(2) A trapezoid $ABCD$, where AB is parallel to CD, is circumscribed around a circle. The sides AD and BC are the diameters of two circles (not shown). Prove that these two circles are tangent, i.e. intersect at exactly one point.



(3) Prove that there does not exist a continuous function $f(x)$ defined for all $x > 0$ such that $f(x)$ is rational if and only if $f(2x)$ is irrational.

(4) A crew of mowers had to mow two meadows, one of which is two times as big as the other. The entire crew spent the first half of the day mowing the larger meadow. In the second half of the day, half of the crew continued mowing the larger meadow and the other half of the crew began to mow the smaller meadow. By the end of the day, the larger meadow was mowed, but a section of the smaller meadow remained unmowed. The next day, one crew member, working the whole day, finished mowing the smaller meadow. How many members are in the crew?

2. Answers

Grades 3-4

(1) C

(2) North

(3) 2100 square centimeters

(4) 3

Grades 5-6

(1) 42

(2) $\frac{5}{8}$

(3) 2018

(4) 36

Grades 7-8

(1) 12

(2) STEVENS = 1930321 and HOBOKEN = 7565432. Therefore, HENS = 7321.

(3) $\frac{1}{8}$

(4) $b > 0$ and $d > 0$

Grades 9-10

(1) 301

(2) 1870 (this is the year in which Stevens was founded!)

(3) $6m$

(4) 6

Grades 11-12

(1) $R_1 = 1$. (The other remainders are $R_2 = 4$ and $R_3 = 10$. $R_2 - R_1$ is a multiple of 3, and $R_3 - R_2$ is a multiple of 6.)

(2) If a circle is inscribed in a quadrilateral, then the sums of the lengths of opposite sides are the same. The line connecting the centers of the tangent circles is the center line of the trapezoid and goes through the point of tangency. Its length is half the sum of the sides AD and BC and therefore equal to the sum of the radii of the two circles.

(3) Assume $f(x)$ is such a function, and consider the functions $g(x) = f(x) + f(2x)$ and $h(x) = f(x) - f(2x)$. Since the sum of a rational number and an irrational number is irrational, $g(x)$ and $h(x)$ take only irrational values and, since they are continuous, are therefore both constant. It follows that the function

$g(x) + h(x) = 2f(x)$ is constant as well, which is a contradiction since it must have both rational and irrational values.

(4) 8