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# Ph.D. DISSERTATION DEFENSE

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| **Candidate:** | Jiajie Lu |
| **Degree:** | Doctor of Philosophy |
| **School/Department:** | Charles V. Schaefer, Jr. School of Engineering and Science / Mathematical Sciences |
| **Date:** | Friday, May 1, 2026 |
| **Time/Location:** | 11:30 a.m. / Martha Bayard Room 203 |
| **Title:** | Statistical Inference on Stochastic Arrangement Increasing and its Lipschitz Generalization |
| **Chairperson:** | Dr. Xiaohu Li, Department of Mathematical Sciences, School of Engineering & Sciences |
| **Committee Members:** | Dr. Darinka Dentcheva, Department of Mathematical Sciences, School of Engineering & Sciences<br>Dr. Benjamin Leinwand, Department of Mathematical Sciences, School of Engineering & Sciences<br>Dr. Sevinn Olafsson, School of Business |

## ABSTRACT

This thesis studies the stochastic arrangement increasing (SAI) property of random vectors and develops new theoretical and statistical tools for its analysis. The SAI property is an important dependence notion in insurance, reliability, and risk management, where it is closely related to the structural form of optimal decisions in stochastic optimization problems. Despite its significance, a general framework for statistical inference on SAI has remained largely undeveloped. This thesis aims to fill that gap while advancing the study of SAI and related dependence structures.

The first part of the thesis investigates the SAI property for skew-normal distributions. Necessary and sufficient conditions are established for the bivariate skew-normal case, and the analysis is further extended to the multivariate setting. This study also motivates a related investigation of the usual stochastic order for skew-normal distributions, yielding new insight into their dependence and ordering structures.

The second part develops a framework for estimation and hypothesis testing of the SAI property. To address the analytical and computational difficulties of direct inference on SAI, a Lipschitz-relaxed version, called L-SAI, is introduced. Under mild regularity conditions, L-SAI is shown to be equivalent to SAI. Based on this relaxation, a discrepancy measure, its empirical counterpart, and a testing procedure are developed.

The final part extends this methodology beyond SAI to a broader class of arrangement-increasing dependence structures. Several existing results are reformulated under the corresponding Lipschitz-relaxed framework, leading to a more unified and practical approach. Overall, this thesis provides both new characterization results for important distribution families and a general inferential framework for analyzing stochastic dependence structures in applications.